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THERMONUCLEAR PLASMA CONTAINMENT IN OPEN-ENDED SYSTEMS

Harold Grad

September 30, 1960

AEC Research and Development Report

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Abstract

A survey is presented of the theory of confinement in open-ended systems (such as mirror machines and cusped geometries) together with the slight amount of experimental confrontation with theory that exists at present.

Thermonuclear Plasma Containment in Open-Ended Systems

A charged particle in a magnetic field is greatly restricted in its motion in the direction perpendicular to H, but it is relatively free to move along a field line. This suggests that the problem of the loss of particles along magnetic field lines may pose special difficulties in a magnetic containment device.

A simple expedient to circumvent this difficulty is to bend the magnetic field lines to form an approximately toroidal figure. As might be expected, such a simple solution is found to introduce a host of new difficulties. Among these, I will mention only:

a) A linear system is cheaper and easier to build and operate.

b) A linear system has a tendency to be more stable.

c) The particles lost through the ends of a linear device are not wasted (at least not in a developmental stage) since they can be very useful in diagnostics to directly sample the contents of the plasma.

d) The theory is much simpler for a typical linear system. This is not merely a lazy theoretician's

excuse to avoid labor! It is exceedingly important in a subject in which reliable experimental data are so difficult to obtain and so hard to interpret, to be able to directly confront experiment with firm theory in order to promote the deeper understanding of both.

There are two general classes of linear configurations. One (typified by the Mirror Machine) is characterized by a magnetic field which passes through the device from one end to the other (this trick is done with mirrors). The other (of which the Picket Fence and Cusped Geometry are notable special cases) has a magnetic field which enters with opposite orientation from the two ends and leaves radially along a mid-section, leaving a relatively low-field region at the center.

The behavior with regard to containment suggests an entirely different morphological classification than is suggested by the geometrical arrangement of the field coils.[1] To evaluate containment, one must know whether there is

a) a relatively large region where the magnetic field strength is small, or

b) appreciable Coulomb scattering within the device.

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1. H. Grad, "Developments in the Theory of Cusped Geometries", TID-7520, p. 148, 1956.

There exists an unfortunate terminology in this subject. There are two methods of computation which are referred to as "cusp loss" and "mirror loss" theories. There are also two generic coil arrangements referred to as "cusp" and "mirror" geometries. Unfortunately, a given theory does not necessarily apply to the geometry of the same name. For example, a high-compression mirror device behaves, with regard to particle losses, more like a cusped geometry than a conventional mirror machine.[2] On the other hand, a general type of cusp-like field configuration can have one part of the apparatus in a cusp-loss regime and another in a mirror-loss regime.[3]

It is advisable to introduce a discussion of the general containment problem in a linear system by the analysis of a sequence of highly idealized special mathematical models. These have been selected to illuminate several distinct physical mechanisms which, in an actual physical system, may act in concert or may typify various limiting cases.

In slide 1 we show a schematic Mirror Machine. The magnetic field is axially symmetric, and there are mirrors,

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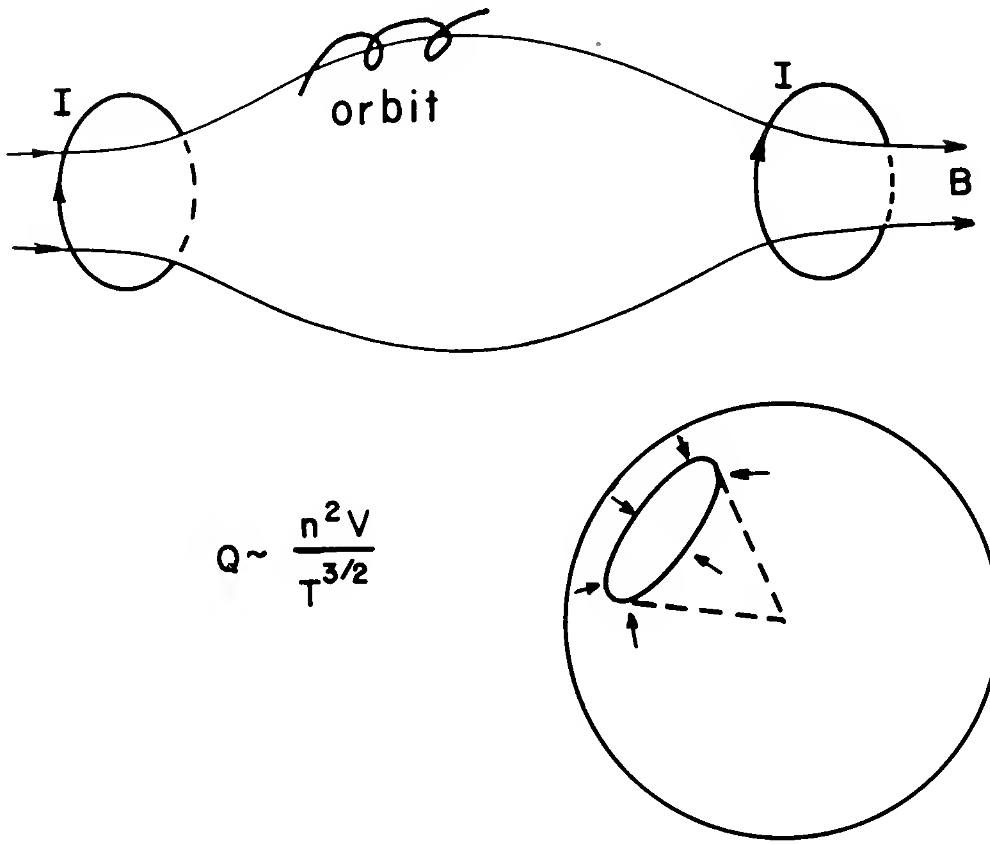
2. A. C. Kolb, H. R. Griem, W. R. Faust, "Dense Plasmas Confined by External Fields", Proc. of Fourth International Conf. on Ionization Phenomena, Uppsala, 1959.

3. H. Grad, footnote 1.

i.e., strengthened field regions, at the two ends. The loss cone analysis of particles in such a device is well-known.[4] Briefly, the loss cone contains those



Slide 1

particles whose velocities are directed within a certain angle of the local magnetic field direction. Insofar as the energy, angular momentum (derived from the axial symmetry) and magnetic moment of a particle are constant to some

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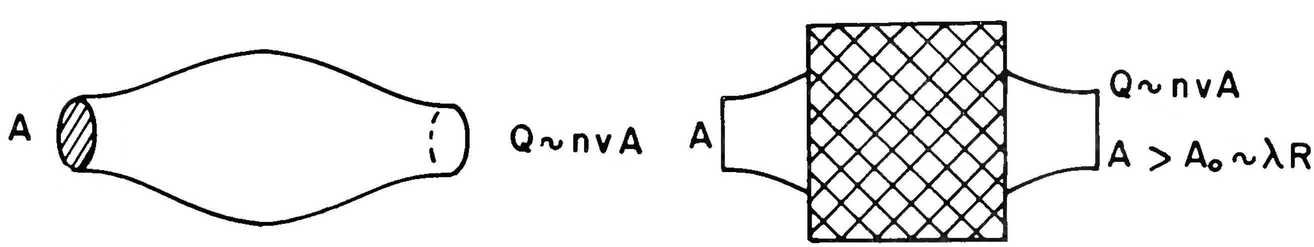
4. R. F. Post, "Summary of UCRL Pyrotron (Mirror Machine) Program", Proc. of the Second International Conference on the Peaceful Uses of Atomic Energy, Vol. 32, p. 245 (Geneva 1958).

approximation, one concludes that particles within the loss cone are lost instantly (within one transit of the device), whereas all other particles are retained indefinitely. Coulomb scattering among the particles will alter the three above-mentioned constants of the motion and will lead to a diffusion of particles into the loss cone and consequent loss from the system. The mean containment time, according to this theory, is closely related to the mean Coulomb scattering time, and one obtains a formula for the rate of loss of particles, Q, as shown in slide 1; (n is the density, T the temperature, V the volume - a slow logarithmic dependence of the loss rate on the mirror ratio has been omitted).

The second model (Figure (a) in Slide 2) considers the plasma to be a classical fluid (say, with small mean-free-path) contained within rigid walls. The loss rate evidently scales as shown; v is a representative particle speed, and A is the geometrical area of the orifice.



(a) (b)

Slide 2

The third model (Figure (b) in Slide 2) is taken to be a long mirror device in which the mean-free-path is smaller than the length of the apparatus but is larger than the dimensions of the mirror region. In order to compute the loss rate, it suffices to replace the central plasma region by a black box which separates the mirrors and serves to randomize magnetic moments. Within the mirror region, a loss cone can be computed using the constant value of the magnetic moment. The black box supplies particles which keep the loss cone full. The loss rate is then given by a formula similar to that for the classical orifice.[5] The principal difference is that the value of the area A to be inserted in this formula is not completely arbitrary. It is found to have a minimum value, A_o, which is the order of the product of a representative Larmor radius by the radius of the plasma in the straight section. The full significance of this "black-box" model will be made clear shortly.

As a fourth model, we take the Picket Fence[6] (Slide 3). This coil configuration was originally suggested as a means of economizing on magnetic energy; its real virtues were

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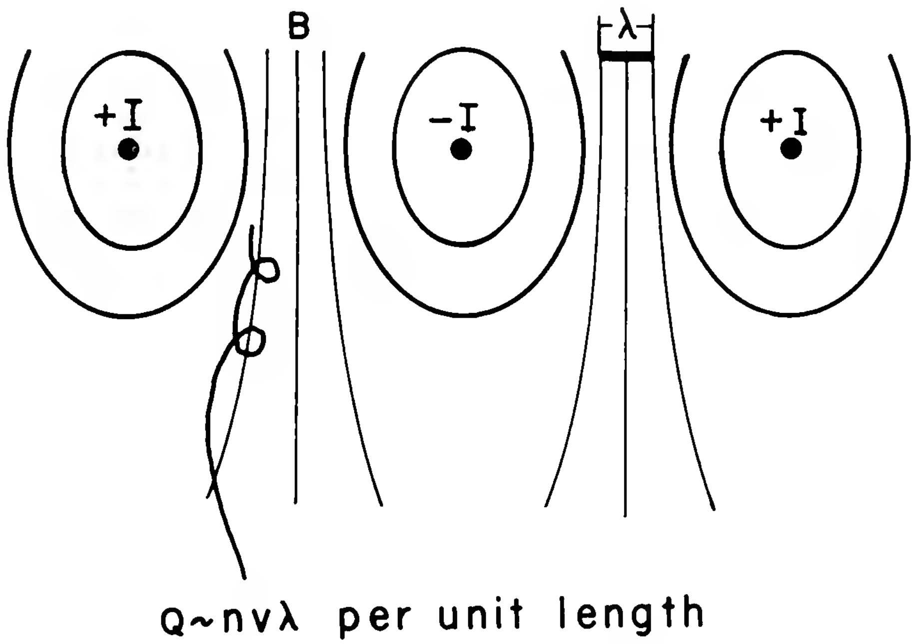
5. H. Grad, footnote 1 and A. C. Kolb et al., footnote 2.

6. J. L. Tuck, "Picket Fence", WASH-184, p. 77 (1955).

discovered later. The coils are a succession of alternating line currents producing a field which decays exponentially



Slide 3

from the "fence". Particle losses from a tenuous plasma which is presumed to be on one side of the fence have been estimated.[7] This computation ignores the possibility of trapped particles and estimates the flow rate of only the particles arriving from infinity. The result is a formula similar to the classical orifice flow, but with an opening comparable in size to the Larmor radius.

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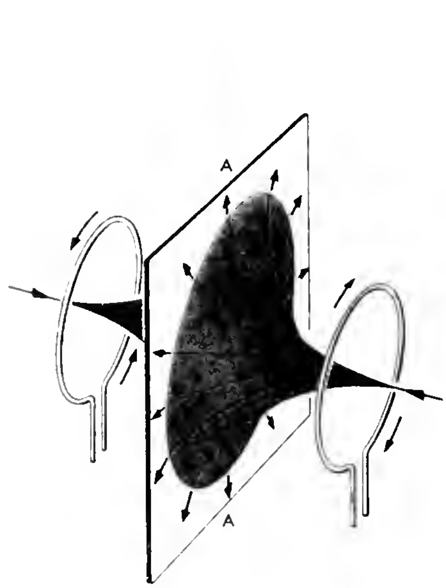
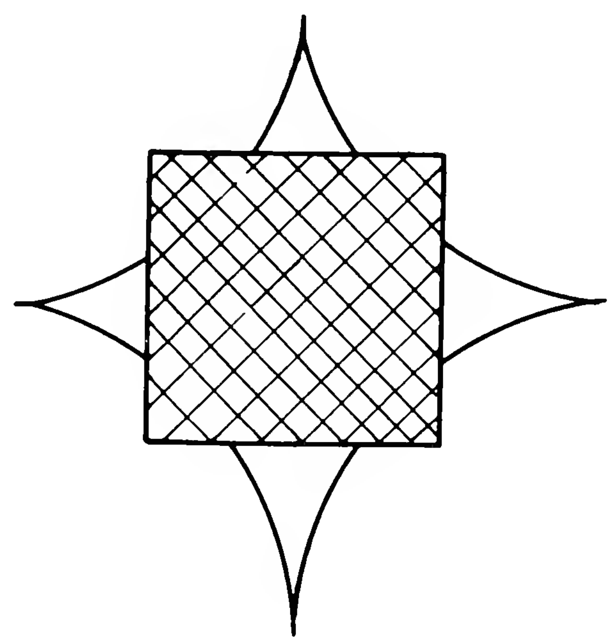
7. Hartland Snyder, private communication, 1954.

The next model to be discussed is the Cusped Geometry (Slide 4). This containment configuration was proposed in 1955 as a result of stability considerations.[8] The plasma is roughly spindle-shaped with two point cusps and a peripheral ring or line cusp. The coil arrangement can be considered to be that of a mirror machine but with currents opposed. The primary feature of this model is that it is postulated that there is complete separation between the plasma region (which has no field) and the vacuum region (without plasma).



Slide 4



Slide 5

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8. H. Grad, "Motion and Stability of Free Surfaces", WASH-289, p. 115, 1955.

The particle loss rate for this model was computed in 1955 by analyzing orbits.[9] It is found that, near a cusp, an adiabatic invariant exists. This is not the magnetic moment but serves to replace it and, in particular, allows one to compute a loss cone. Furthermore, one finds almost complete randomization of the value of this adiabatic invariant on traversing the device from one cusp to the next. Clearly, we can use the black-box argument (Slide 5) and compute losses just as for the black-box model of the mirror machine. Specifically, for the parameter values of interest, the equivalent hole size is in this case on the order of the product of the Larmor radius by a representative dimension of the plasma. This, we recall, was the smallest allowable hole size for the black-box mirror divice.

Returning to the mirror geometry, we see that the black box can hide an active scattering mechanism of particles with one another (as in the original conception), but it can

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9. H. Grad, "Theory of the Cuspidor", TID-7503, p. 319, 1956.

   See also: H. Grad, "Theory of Cusped Geometries I - General Survey", NYO-7969 (1957);

   J. Berkowitz, "Theory of Cusped Geometries II - Particle Losses", NYO-2536 (1959);

   J. Berkowitz, et al., Proc. of the Second International Conference on the Peaceful Uses of Atomic Energy, Vol. 31 p. 171 (Geneva 1958).

also replace a strongly nonadiabatic process as in a mirror machine with a very high mirror ratio[10] (scattering against the magnetic field, if one likes).

We are now in a position to summarize and codify the loss theory. In each of the cases discussed, there was a valid adiabatic invariant near the cusp or mirror which allowed us to analyze particle orbits and compute a loss cone. This leads to the crucial question, how many particles are there in the loss cone? The answer is given by a competition between the depletion of the loss cone by flow out the ends and diffusion into the loss cone throughout the plasma volume. This diffusion is a combination of Coulomb scattering and nonadiabaticity. If the combined diffusion processes are sufficiently active to produce an almost full loss cone, they can be replaced by a black box which isolates the various mirrors or cusps. In the general case the resultant containment can be described in terms of two reference times:

a) the containment time computed as if the loss cone were full

b) the mean diffusion time into the loss cone (combined Coulomb and nonadiabatic processes).

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10. H. Grad, footnote 1.

Specifically, the resultant containment time is the sum of these two reference times. As a rough approximation, one can take the larger of the two.

In some cases it will be necessary to distinguish between different regions of the plasma. It will almost always be necessary to treat ions and electrons separately; we will return to this.

Now let us consider the scaling of the various loss formulas[11] (Slide 6), specifically, the scaling of the appropriate time constants, τ. In the mirror and cusp formulas, we see that where either density or temperature is in the numerator in one expression, it is in the denominator in the other. The formulas could not be more dissimilar!

Mirror losses: $\tau \sim \frac{T^{3/2}}{n}$

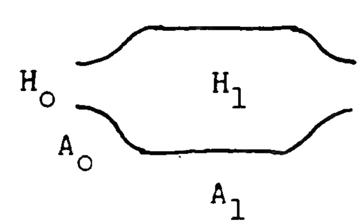
Cusp losses: $\tau \sim \sqrt{\frac{n}{T}}\ A$

Orifice: $\tau \sim \frac{L}{v}\frac{A_1}{A_o}$

Hi Compression Mirror: $\tau \sim \frac{L}{v}\frac{H_o}{H_1}$

L = dimension, A = area

H = field, v = velocity $\sim \sqrt{T}$



Slide 6

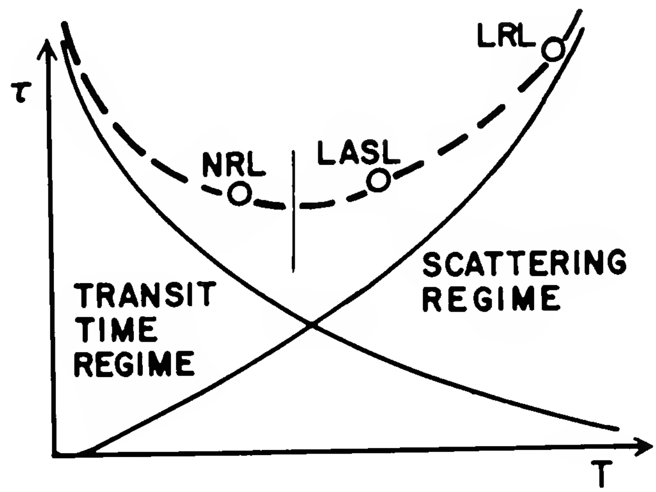
11. H. Grad, NYO-7969, 1957.

Specifically, a conventional mirror device (for which the mirror loss formula is valid, such as at Livermore or at Oak Ridge) should operate at relatively high temperature and low density (at least, from the point of view of containment time). On the other hand, the cusped geometry (for appropriate ranges of parameters) is an inherently low temperature and high density device.

Perhaps the most interesting comparisons can be made of a given device (either cusped or mirror) analyzed with each of the simple theories in turn. In view of the current expreimental situation, the most informative comparison is probably of the two regimes in a mirror device. An intuitive interpretation of the results can be obtained by noticing that the "mirror loss" containment time is essentially the mean Coulomb scattering time, whereas the "high-compression mirror loss" (or black box) containment tims is essentially the transit time modified by the mirror ratio, H_o/H_1.

In Slide 7, we consider a given mirror device with all parameters held fixed except for the temperature. The predicted containment time is sketched according to each theory. The descending solid line is the black box result, the ascending solid line describes the mirror loss formula, and the dashed line is the sum which is the correct

theoretical prediction. The most striking feature of this curve is its minimum which is reached at a temperature where the scattering time is equal to the modified transit time.



Slide 7

Several of the relevant mirror experiments have been located (schematically) on this curve. The Livermore mirror experiments (LRL) are all safely up in the scattering regime; the high temperature DCX experiment is off the diagram on this branch. All of the high-compression mirror experiments are scattered around the minimum; these include the Naval Research Laboratory experiments (NRL) slightly to the left of the minimum and the Los Alamos Scylla devices (LASL) slightly to the right.

In what follows we shall confine the discussion to the more interesting (from the point of view of scaling) high-compression mirror experiments. The first and most striking conclusion is that any change of parameters from the values which are experimentally accessible today cannot help but be an improvement! Before moving, however, one should decide on the direction of motion. Suppose, for the sake of argument, that the LASL intention is to ultimately move to the right (as is indicated by the present location). One should first attempt to raise the temperature and should only increase the density and size when necessary for thermonuclear power production. By the same token, let us assume that the NRL ultimate goal is on the left branch. The last thing to attempt should be an increase in temperature, only when ultimate thermonuclear power production is in sight. First one should attempt to obtain a well-contained laboratory plasma by increasing the size and mirror ratio and then the density.

Suppose these suggestions were to be followed, then what would be the differences in observational techniques in the two regimes? In the high temperature scattering regime, neutron flux would provide a useful index. In the low-temperature transit-time regime, neutron emission would be a very poor guide; one should rather concentrate on the confirmation of the predicted scaling laws with regard to size and mirror ratio. From

the published individual emphasis on diagnostic techniques,[12] one would conclude that the LASL and NRL goals do indeed lie on opposite branches of this curve.

To summarize these consequences of the scaling laws, insofar as our present knowledge is an adequate guide, there may well be two valid and consistent but entirely divergent paths to the thermonuclear goal using the mirror geometry.

I will comment only briefly on the experimental verification of these scaling laws since this topic has been comprehensively reported on other occasions. At Livermore, an extensive series of experiments has confirmed the mirror loss theory dependence on density and on temperature exactly as would be expected in a highly adiabatic device in the scattering regime.[13] At the other extreme, high compression containment experiments at NRL have shown good agreement in length and mirror ratio scaling with the theory that is expected to be applicable there.[14] As for the cusped geometry and picket fence, although these has been a recent upsurge of experiments (notably at General Atomic, Los Alamos, Livermore,

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12. A. C. Kolb, et al., footnote 2. See also W. C. Elmore, E. M. Little, W. E. Quinn "Neutrons from Plasma Compressed by an Axial Magnetic Field (Scylla)", Proc. of Second Internat'l. Conf. on the Peaceful Uses of Atomic Energy Vol. 32, p. 337 (Geneva 1958).

13. R. F. Post, footnote 4.

14. A. C. Kolb et al., footnote 2.

and Stevens in this country and several abroad), it is still too early for proper assessment.

Now we turn again to the cusp-type geometries, this time to look at a practical physical plasma from the point of view of the idealized mirror and cusp loss theories. Even the scaling shows interesting features, but to what extent can we make this theory quantitative?

The first remark is that a given plasma will in general exhibit mirror-like containment in its outer reaches and non-adiabatic cusped containment for those particles with orbits venturing near the low-field center of the device. Also, the point of departure can be entirely different for ions and electrons. Of course, a small mean-free-path makes the plasma homogeneous and wipes out this distinction.

The next complication is that, with both ions and electrons present and strongly interacting, large electrostatic potentials will be created. Furthermore, it can be shown that the precise distribution as well as magnitude of a sheath potential can affect the loss rate enormously. Some work has been done in analyzing this sheath problem and the much more intricate problem of the merging of two sheaths at a cusp, but much more remains to be done.

Now, suppose this sheath problem has been solved. In general, it will turn out that the computed ion and electron

loss rates are different.[15] To offset this, there must exist global charge separation potentials as well as localized sheath potentials. Again a small part of this analysis has been carried out; much more remains to be done. It is difficult to summarize the status of this work briefly; but the simplest general criterion seems to be that for best containment the sheath should be as thin as possible. An important point to keep in mind is that up to two orders of magnitude are at stake in refining this theory (or in achieving this experimentally). There is some theoretical evidence that a plasma will, under certain circumstances, tend to form a desirable thin sheath.

The two complications mentioned above combine to produce an astronomical increase in complexity. Consider the cusp-like magnetic field configuration of Slide 8. There is neither a sharp boundary (cusp loss theory) nor an infinite plasma without trapped orbits (picket fence theory). As a rough estimate, one can guess that there exists a black box region (cross-hatched) of dimensions comparable to the Larmor radius (suitably defined) with the property that orbits which pass inside the black box are strongly nonadiabatic and orbits which lie completely outside are quite adiabatic.

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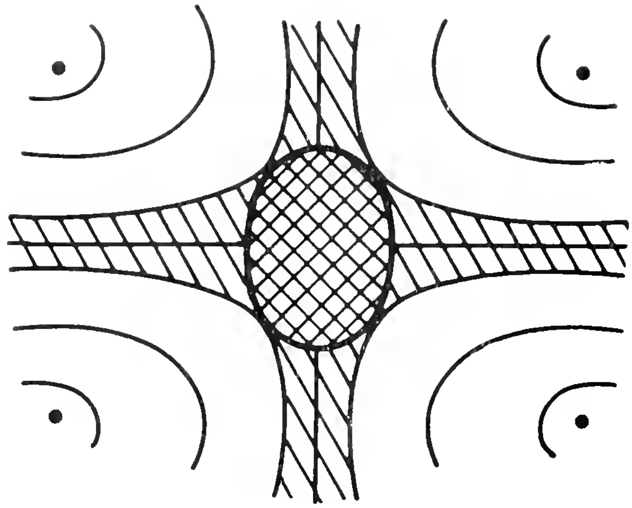
15. Footnote 9.



Slide 8

The former class of particles will be governed by a cusp loss formula and the latter by a mirror loss formula. The two regions differ in the relative loss rates of ions and electrons; therefore a sheath will be set up between them. They also differ in their relative containment of hot and cold particles; the Maxwell tail will tend to be depleted in the inner region and preserved in the outer. If the loss rates of inner and outer portions of the plasma are found to differ considerably, reversed pressure gradients might result causing instability. After reaching roughly this point in the analysis some three years ago, wiser judgement suggested that it was time to look for more rewarding theoretical problems! However, with experiment as a guide, a fresh start seems called for.

A word of caution should be injected here concerning the interpretation of experimental observations of a bare,

unqualified "containment time". As a plasma decays, there is a tendency to shift from the inner (nonadiabatic) to the outer (scattering) regime because of the different density scaling. At the same time, non-Maxwellian effects favoring the bulk of the slow particles in the central region might shift the scaling in the opposite direction, and a shift in sheath potentials could mix things up even more. Finally, a residue of low-temperature high-mass impurities could exhibit spurious long-term cusped containment. All in all, it may be very hard to compare any simple experimental result with a simple containment theory.

Before concluding, some remarks are probably in order to explain how the plasma under discussion ever got into the apparatus. Mirror machine injection has been comprehensively discussed elsewhere.[16] The cusped geometry offers its own unique difficulties and advantages. There are two interesting basic procedures:

a) to independently create a plasma and then apply a cusped field

b) to inject plasma into an already existing cusped field.

In either event, we recall that for reasonable containment of a cusped geometry, we want a dense plasma.

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16. Footnote 4.

While both methods were originally envisioned as theoretical possibilities, the only method that seemed practical in 1955 was injection using the then available plasma rail source.[17] Preliminary experiments of this type were performed in 1956, but with inconclusive results.[18] Shortly later, a technique of shock heating of dense plasmas was developed[19] which would, in principle, allow the application of a cusped field to an existent dense, hot plasma, but this possibility was not exploited. More recently, with the development of greatly improved plasma guns, injection has come to the fore again. In this country, cusped injection experiments under way (or projected) exist at the sites previously mentioned.

There is still a great variety of specific injection techniques. First there is slow injection. By this we mean use of a plasma beam which has an energy density which is so low that it barely influences the external magnetic field. This problem can be analyzed theoretically using individual

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17. W. H. Bostick, Proc. of 9th International Astronautical Congress, Amsterdam 1958, p. 794.

18. D. Finkelstein, G. A. Sawyer, T. F. Stratton, Phys. Fluids 1, 188 (1958).

19. A. C. Kolb, Phys. Rev. 107, 345 (1957).

particle orbit theory.[20] Roughly speaking, a particle is trapped if it passds through the "black box" nonadiabatic region. The more exciting possibility is fast injection of a burst of plasma which is dense enough to deform the magnetic field. This problem is harder to analyze. The black-box effect (i.e., the spreading of the beam) is reduced by the reduction of the magnetic field at entry; depending on the parameters, part of the beam may go right out the other end. One possible expedient for preventing this was indicated by early experiments with colliding plasmoids.[21] Another possibility, recently suggested, is that one might hope to stop the plasma at the other end by strengthening the second mirror field.[22] This is a very difficult problem to analyze theoretically, and there are some experimental indications[23] that this analysis is over-simplified and the technique is unfeasible. However, there are many alternative expedients for improving trapping efficiency[24] and there is no reason at this time to doubt the practicality of some form of trapping.

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20. H. Grad, Phys. Rev. Letters 4, 222 (1960).

21. W. H. Bostick, Phys. Rev. 104, 292 (1956).

22. J. L. Tuck, Phys. Rev. Letters 3, 313 (1959).

23. F. Coensgen, W. Nexsen, W. Cummins, and A. Sherman, Bull. Amer. Phys. Soc. 5, 327 (1960).

24. Footnote 20.

In somewhat tangential conclusion, I would like to comment that it is possible to see a trend toward increasing consideration of the more stable containment configurations. The reason is the almost directly conflicting requirements of magnetic efficiency and stability. As a trend, this may be specious, but the evidence is as follows:

a) In the early days, the pinch looked very promising. This is a beautiful experiment since the magnetic pressure compressing the plasma increases as the pinch develops. By the same token, the increasing magnetic efficiency implies a field gradient toward the plasma, thus great instability.

b) In recent years, there has been considerable excitement about longitudinal magnetic field compression, with or without mirrors. This is marginally stable or not very unstable and is also neutral with regard to magnetic field efficiency.

c) More recently, there has been increased favor in some quarters towards cusped geometries which are at the same time the most stable and in some respects, the most inefficient.

However, it should be remarked that the relatively low density experiments (e.g., Mirror Machine at Livermore and Oak Ridge, Stellerator at Princeton) seem to be less subject to revolutionary changes.

At any rate, the points of contact between theory and experiment seem to be expanding, and particularly, with the growth of experiments in cusped geometries, this is probably the last time that it will be possible to present a nice theoretical talk on this subject, completely unencumbered by any hard experimental facts!

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